LIQUID FLOW IN A LONG CAPILLARY

V. I. Kolesnichenko

UDC 532.68

Addition of an additional force term to the Porkhaev equation is proposed for description of the process of liquid influx into a long capillary.

Capillary influx of liquid involves physicochemical hydrodynamics processes. If we consider this process from a hydrodynamic viewpoint (without consideration of physicochemical features), it can be described by the well-known [1] equation of Prokhaev

$$\frac{d^2l}{dt^2}+\frac{1}{l}\left(\frac{dl}{dt}\right)^2+\frac{8\eta}{r^2\rho}\frac{dl}{dt}-\frac{2\sigma\cos\Theta}{r\rho l}=0. \tag{1}$$

It is assumed here that the capillary is located horizontally, i.e., influx occurs without action of the force of gravity $g\sin\alpha = 0$.

Considering the first two terms of Eq. (1) to be infinitely small, we obtain the well-known expression

$$l=\left(\frac{\sigma r\cos\Theta}{2\eta}t\right)^{1/2}, \tag{2}$$

which is used in models of penetration of capillary-porous bodies. However, this relationship describes the process of liquid influx to a long capillary incorrectly.

Experimental results on displacement of a liquid meniscus in a long ($L \approx 1.3$ m) horizontal capillary tube with radius $r = 4.5\cdot10^{-4}$ m were presented in [2]. Horizontal positioning of the glass tube was monitored; moreover, a portion of the experiments for each liquid were performed after turning the tube through 180°, which eliminated the effect of any possible small inclination of the tube on the experimental results. After each experiment the capillary was dried with compressed air for 10-15 min. A plane vessel into which the liquid studied was poured was attached to one end of the capillary while the other end remained open. The effect of hydrostatic liquid pressure in the vessel on liquid motion in the capillary was eliminated by establishing the level of the surface of a measured quantity of liquid above the capillary input orifice.

Time was measured beginning when the meniscus passed a mark at a distance $\ell_0 = 0.3$ m from the capillary input. Thus, "developed" capillary influx was studied. A stopwatch recorded the time of meniscus passage by marks made on the capillary. The experimental points shown in Fig. 1 do not fit the linear dependence $\ell^2(t)$ which follows from Eq. (2).

This fact apparently indicates that the Porkhaev model does not consider a force term which becomes significant with increase in the length of the wetted segment ℓ. Therefore, we propose that the value of this additional force term is proportional to ℓ. In this case, as will be shown below, the solution of a modified equation (1) completely satisfies the experimental results presented in Fig. 1.

The modified Porkhaev equation has the form

$$\frac{d^2l}{dt^2}+\frac{1}{l}\left(\frac{dl}{dt}\right)^2+\frac{8\eta}{r^2\rho}\frac{dl}{dt}+\frac{l}{\tau^2}-\frac{2\sigma\cos\Theta}{r\rho l}=0, \tag{3}$$

where τ^2 is some empirical proportionality coefficient. From dimensionality considerations this coefficient will have the dimensions of time squared. The initial conditions

Magnetic Dynamics Division, Ural Branch. A. A. Blagonrvavov Machine Study Institute, Academy of Sciences of the USSR, Perm'. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 57, No. 2, pp. 229-232, August, 1989. Original article submitted February 17, 1988.

0022-0841/89/5702-0901$12.50

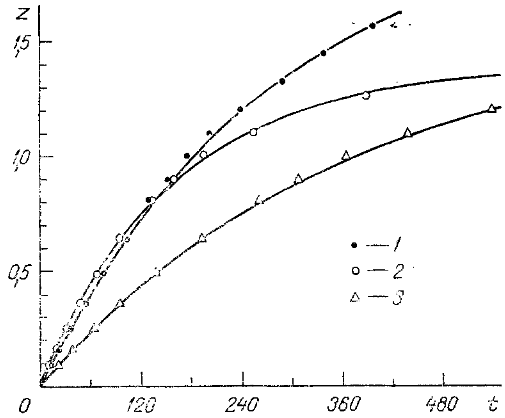

Fig. 1. Square of liquid column length $z = \ell^2$ vs. time t for following substances: 1) distilled water; 2) carbon tetrachloride; 3) ethyl alcohol. z, m^2; t, sec.

$$t = 0, \; l = 0, \quad \frac{dl}{dt} = \left(\frac{2\sigma \cos \Theta}{r\rho}\right)^{1/2}. \tag{4}$$

The initial condition for velocity was obtained from Eq. (3) by performing the limiting transition as $\ell \to 0$.

Making the replacement of variables [3] $z = \ell^2$ and introducing the notation

$$\tau_0 = \frac{r^2\rho}{8\eta}; \; \tau_1^2 = \frac{\tau^2}{2}; \quad a = \frac{4\sigma \cos \Theta}{r\rho}, \tag{5}$$

we obtain

$$\frac{d^2z}{dt^2} + \frac{1}{\tau_0}\frac{dz}{dt} + \frac{1}{\tau_1^2} z = a. \tag{6}$$

Its solution is

$$z = \frac{a}{k_1 k_2}\left(1 + \frac{k_2}{k_1 - k_2}\exp\{k_1 t\} + \frac{k_1}{k_2 - k_1}\exp\{k_2 t\}\right), \tag{7}$$

where k_1 and k_2 are roots of the corresponding characteristic quadratic equation:

$$k_{1,2} = -\frac{1}{2\tau_0}\left\{1 \pm \left[1 - 4\left(\frac{\tau_0}{\tau_1}\right)^2\right]^{1/2}\right\}, \tag{8}$$

having the properties

$$k_1 k_2 = \frac{1}{\tau_1^2}; \quad k_1 + k_2 = -\frac{1}{\tau_0}. \tag{9}$$

Skipping ahead somewhat, we may note that Eq. (8) contains a small parameter $(\tau_0/\tau_1)^2 \ll 1$. This permits simplification of Eq. (8)

$$k_1 = -\frac{\tau_0}{\tau_1^2}; \quad k_2 = \frac{\tau_0}{\tau_1^2} - \frac{1}{\tau_0}. \tag{10}$$

It can easily be shown that k_2, like k_1, has a negative value. Moreover:

$$\frac{k_1}{k_2} \approx \left(\frac{\tau_0}{\tau_1}\right)^2 + \left(\frac{\tau_0}{\tau_1}\right)^4 \ll 1. \tag{11}$$

Using the latter inequality, we estimate the coefficients of the exponentials in Eq. (7):

$$\frac{k_2}{k_1 - k_2} \approx -1; \quad \frac{k_1}{k_2 - k_1} \ll 1. \tag{12}$$

TABLE 1. Values of a, τ_1^2, τ_0 for Materials Studied

Material	a, m^2/sec^2	τ_1^2, sec^2	$\tau_0 \cdot 10^2$ sec
H_2O	0,299	6,83	2,52
C_2H_6O	0,255	6,21	1,66
CCl_4	0,212	6,61	4,17

Since $|k_1| \ll |k_2|$ [Eq. (11)] and both roots are negative, the value of the second exponential in Eq. (7) is significantly smaller than that of the first. Considering this, together with Eqs. (9), (10), (12), we finally obtain

$$z = a\tau_1^2 \left(1 - \exp\left\{-\frac{\tau_0}{\tau_1^2} t\right\}\right) \tag{13}$$

or, transforming to the original variables of Eq. (5):

$$l = \tau \left[\frac{2\sigma\cos\Theta}{r\rho}\left(1 - \exp\left\{-\frac{r^2\rho}{4\eta\tau^2} t\right\}\right)\right]^{1/2}. \tag{14}$$

For small times, expanding the exponential in a series, we obtain Eq. (2). Thus, in the initial stage of influx the additional force term introduced does not manifest itself. On the other hand, it plays a significant role at large times. While Eq. (2) indicates that liquid in a long horizontal capillary can move practically indefinitely, it follows from Eq. (14) that there exists a finite limit to such flow:

$$l^* \equiv l|_{t\to\infty} = \tau\left(\frac{2\sigma\cos\Theta}{r\rho}\right)^{1/2}. \tag{15}$$

Thus, ℓ^* is a characteristic linear dimension. Using Eq. (5), we may introduce velocity $v = (a/2)^{1/2}$. Then from Eq. (15) we obtain

$$\tau = \frac{l^*}{v} = l^*\left(\frac{r\rho}{2\sigma\cos\Theta}\right)^{1/2}, \tag{16}$$

i.e., the empirical coefficient τ introduced here is some characteristic time, over which the liquid traverses a limiting distance ℓ^*, moving with a velocity v.

Making use of Eq. (16), we rewrite Eq. (3) in the form

$$\frac{d^2l}{dt^2} + \frac{1}{l}\left(\frac{dl}{dt}\right)^2 + \frac{8\eta}{r^2\rho}\frac{dl}{dt} = \frac{2\sigma\cos\Theta}{r\rho l}\left[1 - \left(\frac{l}{l^*}\right)^2\right]. \tag{17}$$

The right-hand term of this expression indicates how the decrease in driving force for the capillary influx occurs as the length of the wetted segment ℓ increases. Such a change can be explained, for example, by the presence of a dependence of dynamic wetting angle on ℓ.

Finally, we will consider an approximation usually made in such problems. In Eq. (6) we drop the term d^2z/dt^2, i.e., we will neglect the inertial force. Solution of the equation obtained with the initial condition t = 0, z = 0 coincides precisely with Eq. (13), the solution of the complete differential equation (6) after simplifications.

Table 1 presents values obtained by processing experimental results, while Fig. 1 shows curves calculated with Eq. (13) and compares them with experimental points, averaged over 8-10 experimental values. The good agreement of the experimental and theoretical results is evident. It follows from the data of Table 1 that the inequality assumed above, $(\tau_0/\tau_1)^2 \ll 1$, is valid for all the material tested.

Thus, introduction of an additional force term into the Porkhaev equation is quite effective for description of liquid flow in a long capillary.

NOTATION

ℓ, length of liquid column in capillary at time t; r, capillary radius; L, capillary length; η, liquid dynamic viscosity coefficient; σ, liquid surface tension coefficient; ρ, liquid density; θ, static wetting angle; g, acceleration of gravity; α, angle of inclination of the capillary to the horizontal; τ, empirical proportionality coefficient; z, v, τ_0, τ_1, a, ℓ^*, new variables; k_1, k_2, roots of characteristic quadratic equation.

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INFLUENCE OF A POSITIVE PRESSURE GRADIENT ON THE CHARACTERISTICS OF A TURBULENT BOUNDARY LAYER

V. V. Zyabrikov

UDC 532.526

Based on a systematic analysis of present day experimental data published in the literature, a modified Prandtl—Clauser turbulence model is presented which makes it possible to take into account the effect of a positive pressure gradient on the average characteristics of a turbulent boundary layer.

Determination of the characteristics of a turbulent boundary layer subject to the action of a positive pressure gradient constitutes a difficult problem from the experimental point of view. Proceedings of the Stanford Conference of 1980/81 (see [1]) show that there is as yet no full set of published experimental data that exhausts this problem (especially as far as the region before separation is concerned). According to the valid opinion of the authors of [2, 3], the difficulty in making an experimental study of the region close to the point where the turbulent boundary layer separates is associated with the emergence of short-duration reverse flows at a significant distance from the "stationary" separation point and with the need for using measuring instruments sensitive to the direction of the rate of flow. Although the first paper on this theme appeared in 1968 (see [2]), it is only recently that sufficiently detailed results of systematic measurements have been published [4-6] that justify modification of the Prandtl-Clauser model of turbulence. The present research was conducted under the guidance of L. G. Loitsyanskii.

Distribution of Longitudinal Velocity and Frictional Stress in the Interior Region of a Turbulent Boundary Layer. By the interior region of a turbulent boundary layer we mean that portion of it in which the turbulent viscosity increases with increasing distance from the wall. In contrast to the exterior region the interior region depends weakly on the prehistory of the flow and possesses a relative autonomy: The characteristics of this region can be regarded as functions only of the parameters of pressure gradient $p_* = (\nu/\rho)(dp/dx)/v_*^3$ and convective acceleration $g_* = \nu(dv_*/dx)v_*^2$ [7]. The interior region of the turbulent boundary layer with a positive pressure gradient consists of a viscous sublayer, a transitional portion, a logarithmic region, and a half-power law subregion [8]. The problem of determining the damping factor in the transition section was examined in detail in [7-9]. For a positive pressure gradient of arbitrary magnitude the damping factor can be approxi-

M. I. Kalinin Leningrad Polytechnical Institute, Leningrad. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 57, No. 2, pp. 232-239, August, 1988. Original article submitted February 18, 1988.

0022-0841/89/5702-0904$12.50